sparselizard

the user friendly

finite element

c++ library

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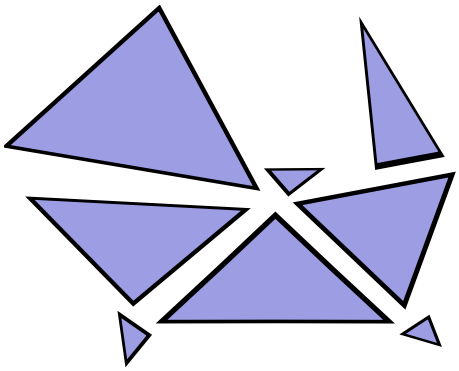

Contents

1 What is sparselizard

Sparselizard (Copyright (C) 2017- Alexandre Halbach and Christophe Geuzaine, University of Liege, Belgium) is an open source C++ finite element library provided under the terms of the GNU General Public License (GPL), version 2 or later. It is meant to be user friendly and concise while still fast and multithreaded. A working example solving an electrostatic problem on a 3D disk with volume charges and grounded at its boundary is shown below.

```
int vol = 1, sur = 2;             // Disk volume and boundary as set in 'disk.geo'
mesh mymesh("disk.msh");          // The elements in the GMSH mesh can be curved!

field v("h1");                    // Nodal shape functions for the electric potential
v.setorder(vol, 4);               // Interpolation order 4 on the whole domain
v.setconstraint(sur, 0);          // Force 0 V on the disk boundary

formulation elec;                 // Electrostatics with 1 nC/m^3 charge density
elec += integral(vol, -8.85e-12 * grad(dof(v)) * grad(tf(v)) + 1e-9 * tf(v));
elec.generate();
vec solv = solve(elec.A(), elec.b());

v.setdata(vol, solv);             // Transfer data from solution vector to v field
(-grad(v)).write(vol, "E.pos");   // Write the electric field
```

Sparselizard can handle a rather general set of problems in 1D, 2D and 3D such as mechanical, acoustic, thermal, electric, magnetic, fluid, electromagnetic, piezoelectric,... problems (provided in form of a weak formulation as detailed in `https://en.wikipedia.org/wiki/Weak_formulation`). Multiphysics problems, nonlinear problems or nonlinear multiphysics problems can be simulated as well. The problems can be readily solved in time with a time-stepping resolution or with the natively supported multiharmonic resolution method. In the latter case the steady-state solution of a time-periodic problem can be obtained in a single step, for linear as well as for nonlinear problems. The library comes with hierarchical high order shape functions so that high order interpolations can be used with an interpolation order adapted to every unknown field and geometric region.
Built-in geometry definition and meshing tools are provided for simple geometries. Complex geometries can be meshed with the open-source GMSH meshing software (`www.gmsh.info`). Points, lines, triangles, quadrangles, tetrahedra, hexahedra, prisms or any combination thereof is accepted in the mesh. The result files output by sparselizard are in .pos format supported by GMSH.

Sparselizard has been successfully tested on Linux, Mac and Windows. Working examples can be found in the **examples folder** in the project.

2 How to install sparselizard

Sparselizard can be obtained at the following address:

`www.sparselizard.org`

either as a static library that can be readily used or as source code to compile as detailed below.

Before compiling sparselizard, the external libraries below must be installed. For that make sure you have git and the gcc, g++ and the **standard** gfortran compilers. On Ubuntu linux install them with:

```
sudo apt-get install git
sudo apt-get install gfortran
sudo apt-get install gcc
sudo apt-get install g++
```

Once the compilers are available the required external libraries must be installed. This can be done easily by running in the provided order all bash scripts in folder 'install_external_libs'. Each script installs with the right options the corresponding external library in the 'SLlibs' folder in the home directory. In case this does not work for a given library, please install it yourself with the configuration options detailed in the bash script. In case you do not want to use the standard installation directory or want to use an already available library do not forget to change the library path accordingly in the makefile and in 'run_sparselizard.sh'.

The external libraries used are the following:

- OpenBLAS: is used for optimised and multithreaded operations on dense matrices and vectors. More information at `www.openblas.net`.
- PETSc: in combination with MUMPS is mainly used to solve the large sparse algebraic problems. More information at `www.mcs.anl.gov/petsc` and `mumps.enseeiht.fr`.
- SLEPc: in combination with PETSc is used to solve eigenvalue problems involving large sparse algebraic matrices. More information at `slepc.upv.es`.

Once all external libraries are successfully installed sparselizard can be compiled by simply running 'make' or 'make -j4' if you have 4 computing cores.

3 How to use and run sparselizard

One way of using sparselizard is with the following steps:

1. Edit the 'sparselizard' function in the 'main.cpp' file for your simulation.
2. Run make in the terminal. This should be much quicker this time since only the 'main.cpp' file has to be recompiled.
3. Run your simulation by entering './run_sparselizard.sh' in the terminal.

As an example let us simulate the static deflection of a mechanical disk with some volume force applied to it. This requires to have the original 'main.cpp' and 'disk.geo' files that are available after having downloaded the sparselizard project. This also requires the binary of the open source GMSH meshing software that can be downloaded at `www.gmsh.info`.
Copy the binary to the sparselizard folder then mesh the 'disk.geo' geometry by running './gmsh disk.geo -3' (3 because it is a 3D problem) or with './gmsh disk.geo' to mesh graphically. This creates a 'disk.msh' file which contains the mesh. Now run './run_sparselizard.sh' in the terminal. This runs the code in 'main.cpp' that has just been compiled.
The last step has created the 'u.pos' output file, which gives the exaggerated displacement of the top surface in the thin cylinder geometry when the sides are clamped and a volume force is applied downwards. Open it with './gmsh u.pos'. You don't see anything or it is rough? Don't worry, this is just because the simulation was performed using few hexahedra in the mesh but with an order 2 interpolation! To visualise high order interpolations in GMSH do this:

- Double click in the middle of the window then select 'All view options' at the bottom of the box that appeared. Go to the 'General' tab and tick the 'Adapt visualization grid' box.
- Set 'Target visualization error' to the smallest possible and 'Maximum recursion level' to 3 then press enter. Now you have a finer solution!
- Since the solution is a mechanical displacement you might want to see the (exaggerated) deflection in 3D by double clicking in the middle of the window then selecting 'View vector display' >> 'Displacement' with factor 1.
- In case you see strange lighting effects double click in the middle of the window then select 'All view options' at the bottom of the box that appeared, go to the 'Color' tab and untick the 'Enable lighting' box.

Figure 1 is what you should see. Congratulations for your first simulation with the sparselizard library! In the 'examples' folder you will find more sparselizard examples.

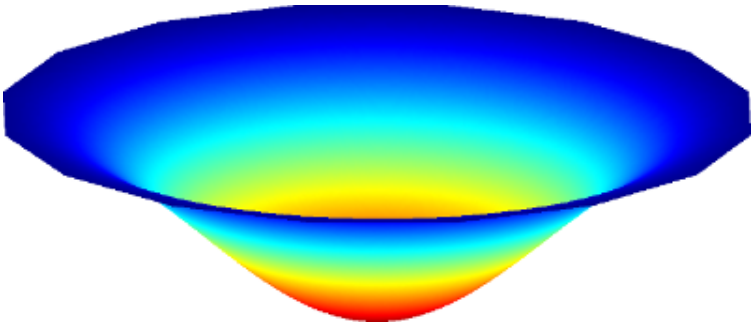

Figure 1: Exaggerated deflection of the 3D disk

4 Objects and functions available in the library

Much of sparselizard is written in C++ in an object-oriented way. It should thus be no surprise that most of the code you write consists in creating and managing objects. Below is the list of the main objects (and namespaces) that you can use in your simulations. In any case we recommend to follow the examples in the **examples folder** to get used to sparselizard.

4.1 The *densematrix* object (/src/densematrix.h):

The densematrix object stores a row-major array of doubles that corresponds to a dense matrix. Only the functions required to use the other objects are detailed.

`densematrix(int numberofrows, int numberofcolumns)`

densematrix B(2,3);

This creates an all zero matrix with 2 rows and 3 columns.

`densematrix(int numberofrows, int numberofcolumns, double initvalue)`

densematrix B(2,3, 12);

This creates a matrix with 2 rows and 3 columns. All entries are assigned to value 'initvalue'.

`densematrix(int numberofrows, int numberofcolumns, const std::vector<double> valvec)`

densematrix B(2,3, {1,2,3,4,5,6});

This creates a matrix with 2 rows and 3 columns. The entries are assigned to the values of 'valvec'. As an example B at row 0, column 2 is set to 3.

`densematrix(int numberofrows, int numberofcolumns, int init, int step)`

densematrix B(2,3, 0,1);

This creates a matrix with 2 rows and 3 columns whose values increase by steps of 'step' (first is 'init').

`int countrows(void)`

densematrix B(2,3);
int numrows = B.countrows();

This counts the number of rows in the dense matrix (2 here).

`int countcolumns(void)`

densematrix B(2,3);
int numcols = B.countcolumns();

This counts the number of columns in the dense matrix (3 here).

`void print(void)`

densematrix B(2,3, 0,1);
B.print();

This prints the matrix values to the console.

`void printsize(void)`

densematrix B(2,3);
B.printsize();

This prints the matrix size to the console.

`double* getvalues(void)`

densematrix B(2,3);
double* vals = B.getvalues();

This returns a double array pointer that lets you read or write any value to the array (stay in the array to avoid a segmentation fault). The array is deallocated when the matrix goes out of scope.

4.2 The *expression* object (/src/expression/expression.h):

The expression object holds a mathematical expression made of $+ - */$ operators, fields, parameters, square operators, abs operators...

`expression(void)`

expression myexpression;

This creates an empty expression object.

`expression(int numrows, int numcols, std::vector<expression>)`

```
mesh mymesh("disk.msh");
field v("h1");
expression myexpression(2,3,{12,v,v*(1-v),3,14-v,0});
myexpression.print();
expression myexpressionsym(3,3,{1,2,3,4,5,6});
myexpressionsym.print();
expression myexpressiondiag(3,3,{1,2,3});
myexpressiondiag.print();
```

The first call creates a 2×3 sized expression array (2 rows, 3 columns) and sets every entry to the corresponding expression in the expression vector. The expression vector is row-major. As an example entry (1,0) in the created expression is set to 3 and entry (1,2) to 0.
The second call creates a symmetric expression array where the provided vector corresponds to the lower triangular part (top to bottom and left to right ordered).
The third call creates a diagonal expression array where the diagonal is filled with the expressions provided in the input vector.

`expression(expression condexpr, expression exprtrue, expression exprfalse)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2, top = 3;
field x("x"), y("y");
expression expr(x+y, 2*x, 0);
expr.print();
expr.write(top, "conditionalexpr.pos");
```

This creates a conditional expression. If the first argument is greater or equal to zero then the expression is equal to the expression provided as second argument. If smaller than zero it is equal to the third argument expression.

`int countrows(void)`

```
expression myexpression(2,1,{0,1});
int numrows = myexpression.countrows();
```

This counts the number of rows in an expression (here 2).

```
int countcolumns(void)
```

```
expression myexpression(2,1,{0,1});
int numcolumns = myexpression.countcolumns();
```

This counts the number of columns in an expression (here 1).

```
void reorderrows(std::vector<int> neworder)
```

```
expression expr (2,2,{0,1,2,3});
expr.print ();
expr.reorderrows({1,0});
expr.print ();
```

This reorders the rows of a matrix expression.

```
void reordercolumns(std::vector<int> neworder)
```

```
expression expr (2,2,{0,1,2,3});
expr.print ();
expr.reordercolumns({1,0});
expr.print ();
```

This reorders the columns of a matrix expression.

```
std::vector<double> max(int physreg, int refinement, std::vector<double> xyzrange = {})
std::vector<double> min(int physreg, int refinement, std::vector<double> xyzrange = {})
```

```
mesh mymesh("disk.msh");
int vol = 1;
field x("x");
std::vector<double> maxdata = (2*x).max(vol, 1);
std::vector<double> maxdatainbox = (2*x).max(vol, 5, {-2,0, -2,2, -2,2});
```

This gives the max/min value of the expression over the geometric region 1. The search of the max/min can be restricted to a box delimited by the last argument (optional) whose form is {xboxmin xboxmax, yboxmin, yboxmax, zboxmin, zboxmax}. The output is {maxvalue, xcoordmax, ycoordmax, zcoordmax}.
The max/min is obtained by splitting all elements 'refinement' times in each direction. Increasing the refinement will thus lead to a more accurate max/min value, but at an increasing computational

cost. The max/min value is exact when the refinement nodes added to the elements correspond to the position of the max/min. For a first order nodal shape function interpolation on a mesh that is not curved the max/min is always exact to machine precision.

```
std::vector<double> max(int physreg, expression meshdeform, int refinement,
std::vector<double> xyzrange = {})
std::vector<double> min(int physreg, expression meshdeform, int refinement,
std::vector<double> xyzrange = {})
```

...
field u("h1xyz");
std::vector<double> maxdatadeformedmesh = (2*x).max(vol, 1);

Same as the previous function but the expression is evaluated on the geometry deformed by field u (possibly curved mesh). The max/min location and delimiting box are on the undeformed mesh.

```
void interpolate(int physreg, std::vector<double>& xyzcoord,
std::vector<double>& interpolated, std::vector<bool>& isfound)
```

mesh mymesh("disk.msh");
int vol = 1;
field x("x"), y("y"), z("z");
std::vector<double> xyzcoord = {0.5,0.6,0.05, 0.1,0.1,0.1};
std::vector<double> interpolated;
std::vector<bool> isfound;
array3x1(x,y,z).interpolate(vol, xyzcoord, interpolated, isfound);
printvector(interpolated);

This interpolates the expression at multiple points whose {x, y, z} coordinates are provided in 'xyzcoord'. The flattened interpolated expression values are concatenated for every interpolation point and put in 'interpolated'. If the ith point was not found in the elements of the physical region 'physreg' then 'isfound[i]' is set to false. For curved elements there is no convergence guarantee for the Newton-Raphson algorithm called by the function.

```
void interpolate(int physreg, expression meshdeform, std::vector<double>& xyzcoord,
std::vector<double>& interpolated, std::vector<bool>& isfound)
```

...
field u("h1xyz");
array3x1(x,y,z).interpolate(vol, u, xyzcoord, interpolated, isfound);
printvector(interpolated);

Same as the previous function but the expression is computed on the mesh deformed by field 'u'.

`std::vector<double> interpolate(int physreg, const std::vector<double> xyzcoord)`

```
mesh mymesh("disk.msh");
int vol = 1;
field x("x"), y("y"), z("z");
std::vector<double> interpolated = array3x1(x,y,z).interpolate(vol, {0.5,0.6,0.05});
printvector(interpolated);
```

This interpolates the expression at a single point whose {x, y, z} coordinates are provided as argument. The flattened interpolated expression values are returned if the point was found in the elements of the physical region 'physreg'. If not found an empty vector is returned. For curved elements there is no convergence guarantee for the Newton-Raphson algorithm called by the function.

`std::vector<double> interpolate(int physreg, expression meshdeform, const std::vector<double> xyzcoord)`

```
...
field u("h1xyz");
interpolated = array3x1(x,y,z).interpolate(vol, u, {0.5,0.6,0.05});
printvector(interpolated);
```

Same as the previous function but the expression is computed on the mesh deformed by field 'u'.

`double integrate(int physreg, int integrationorder)`

```
mesh mymesh("disk.msh");
int vol = 1;
expression myexpression = 12.0;
// Mesh with curved elements at order 3 to accurately
// capture the cylinder geometry and get value 3.7699!
double integralvalue = myexpression.integrate(vol, 4);
```

This integrates the expression over the geometric region 1. The integration is exact for up to 4th order polynomials.

`double integrate(int physreg, expression meshdeform, int integrationorder)`

```
...
field u("h1xyz");
double integralvaluedeformedmesh = myexpression.integrate(vol, u, 4);
```

This integrates the expression over the geometric region 1. The integration is exact for up to 4th order polynomials. It is performed on the geometry deformed by field u (possibly curved mesh).

```
void write(int physreg, int numfftharms, std::string filename, int lagrangeorder = 1)
```

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz"), v("h1 ",{1,2,3});
(u*u).write(vol, 10, "order3.pos", 3);
abs(v).write(vol, 10, "order1.pos");
```

This writes an expression to a file, here with either an order 3 interpolation or, in the latter case with a default order 1 interpolation. The 10 means the expression is treated as multiharmonic, nonlinear in the time variable and an FFT is performed to get the 10 first harmonics. All harmonics whose magnitude are above a threshold are saved with the '_harm i' extension (except the time-constant harmonic).

```
void write(int physreg, int numfftharms, expression meshdeform, std::string filename,
int lagrangeorder = 1)
```

```
...
abs(u).write(vol, 10, u, "u.pos", 2);
```

Same as above but here the expression is evaluated and written on a mesh deformed by field u (possibly curved mesh).

```
void write(int physreg, std::string filename, int lagrangeorder = 1,
int numtimesteps = -1)
```

```
...
(1e8*u).write(vol, "uorder1.pos");
(1e8*u).write(vol, "uorder3.pos",3);
(1e8*u).write(vol, "uintime.pos", 2, 50);
```

Same as two above except that here no FFT is computed. In case the expression is nonlinear and multiharmonic the FFT is required and an error is thus thrown. If 'numtimesteps' is set to a positive value n then the (multiharmonic) expression is saved at n equidistant timesteps in the fundamental period and can then be visualised in time.

```
void write(int physreg, expression meshdeform, std::string filename,
int lagrangeorder = 1, int numtimesteps = -1)
```

```
...
(1e8*v).write(vol, u, "uintime.pos", 2, 50);
```

Same as above but here the expression is evaluated and written on a mesh deformed by field u (possibly curved mesh).

```
void streamline(int physreg, std::string filename,
const std::vector<double>& startcoords, double stepsize, bool downstreamonly = false)
```

```
int vol = 1;
mesh mymesh("disk.msh");
field x("x"), y("y"), z("z");
std::vector<double> startcoords(12*3,0.05);
for (int i = 0; i < 12; i++)
        startcoords[3*i+0] += 0.1+0.05*i;
array3x1(y+2*x,-y+2*x,0).streamline(vol, "streamlines.pos", startcoords, 1.0/100.0);
```

This follows and writes to disk all paths tangent to the expression vector that are starting at a set of points whose x, y and z coordinates are provided in 'startcoords' (these coordinates can for example be obtained via .getcoords() on a shape object). A fourth order Runge-Kutta algorithm is used ('stepsize' is related to the distance between two vector direction updates, decrease it to more accurately follow the paths). The paths will be followed as long as they remain in physical region 'physreg'. In case the vector norm is zero somewhere on the paths or a path is a closed loop the function might enter in an **infinite loop** and never return.
To use this function on closed loops (for example to get the magnetic field lines of a permanent magnet) a solution is to break the loops by excluding the permanent magnet domain from the physical region (the 'regionexclusion' function can be called for that) and set the starting coordinates on the boundary of the magnet.

```
std::vector<double> shapecut(int physreg, shape myshape, std::string filename = "")
```

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz"), x("x"), y("y");
shape quad("quadrangle", -1, {-1,0,0, 1,0,0, 1,0,0.1, -1,0,0.1}, {20,10,20,10});
std::vector<double> vals = (2*x).shapecut(vol, quad, "cut.pos");
```

This interpolates any expression on the mesh nodes of a shape. Only the data points on physical region 'physreg' are returned/saved to the file. If 'filename' is not empty the data is also saved to disk (in that case the expression must be a column vector with 1 to 3 components).

```
std::vector<double> shapecut(int physreg, expression meshdeform, shape myshape
std::string filename = "")
```

...

std :: vector<double> valsdef = (2∗x).shapecut(vol, u, quad);

Same as above but here the expression is evaluated and written on a mesh deformed by field u (possibly curved mesh).

`void reuseit(bool istobereused = true)`

expression myexpression(12.0);
myexpression.reuseit ();

In case you have an expression that appears multiple times e.g. in a formulation and requires too much time to be computed you can 'reuse' that expression. With this the expression will only be computed once to assemble a formulation block and reused as long as it is impossible that its value has changed.

`bool isscalar(void)`

expression myexpression(12.0);
bool a = myexpression.isscalar ();

True if the expression is a scalar (i.e. has a single row and column).

`bool iszero(void)`

expression myexpression(2,1,{0,0});
bool a = myexpression.iszero();

True if the expression is zero (here it is true).

`vec atbarycenter(int physreg, field onefield)`

```
mesh mymesh("disk.msh");
int vol = 1;
field barycentervaluefield ("one"), x("x");
vec myvec = (12∗x).atbarycenter(vol, barycentervaluefield );
barycentervaluefield .setdata(vol, myvec);
(12∗x).write(vol, "expression.pos");
barycentervaluefield .write(vol, "barycentervalues.pos");
```

This outputs a vector whose structure is based on the argument field and which contains the barycenter values of the expression.

`vec integrateonelements(int physreg, field onefield, int integrationorder)`

```
mesh mymesh("disk.msh");
int vol = 1;
```

```
field  integralvaluefield ("one"), x("x");
vec myvec = (10*x).integrateonelements(vol, integralvaluefield , 4);
integralvaluefield .setdata(vol, myvec);
(10*x).write(vol, "expression.pos");
integralvaluefield .write(vol, "integralvalues.pos");
```

This outputs a vector whose structure is based on the argument field and which contains the integral (on every element in region 'physreg' separately) of the expression.

`void print(void)`

```
expression myexpression(12.0);
myexpression.print();
```

Prints the expression to the console.

`expression at(int row, int col)`

```
expression myexpression(2,2,{1,2,3,4});
expression myentry = myexpression.at(0,1);
```

This returns the entry at the requested row and column (here it returns expression 2).

`std::vector<double> evaluate(std::vector<double>& xcoords, std::vector<double>& ycoords, std::vector<double>& zcoords)`

```
field x("x"), y("y"), z("z");
expression expr = 2*x+abs(y)/z;
std::vector<double> xcoords = {0.0, 1.0};
std::vector<double> ycoords = {-2.0, 3.0};
std::vector<double> zcoords = {4.0, 5.0};
std::vector<double> vals = expr.evaluate(xcoords, ycoords, zcoords);
printvector(vals);
```

This evaluates an expression at given x, y and z coordinates.
Only operators and the x, y and z fields are allowed in the expression.

`operators + - * / :`

```
mesh mymesh("disk.msh");
int vol = 1;
expression expr(12.0);
parameter E;
E|vol = 10;
```

```
double dbl = -3.2;
field  v("h1");

expression plus = expr + E + dbl + v;
expression minus = -expr - E - dbl - v;
expression product = expr * E * dbl * v;
expression divided = expr / E / dbl / v;

expression mixed = expr * (-2.0 * v) - E / dbl;
```

The sum, difference, product and division between any two of 'expression', 'field', 'double' and 'parameter' is allowed.

4.3 The *field* object (/src/field/field.h):

The field object holds the information of the finite element fields. The field object itself only holds a pointer to a 'rawfield' object.

`field(std::string fieldtypename)`

mesh mymesh("disk.msh");
field v("h1");

This creates a field v with nodal (i.e. "h1") shape functions.

The full list of shape functions that are available is:

- Nodal shape functions "h1" e.g. for the electrostatic potential or acoustic pressure field.
- Two-components nodal shape functions "h1xy" e.g. for 2D mechanical displacements.
- Three-components nodal shape functions "h1xyz" e.g. for 3D mechanical displacements.
- Nedelec's edge shape functions "hcurl" e.g. for the electric and magnetic fields in the E-formulation of electromagnetic wave propagation (here order 0 is allowed).
- "q6" ("q6xy" or "q6xyz") shape functions (only for rectangular elements) identical to order one "h1" ("h1xy" or "h1xyz") shape functions except that there are two extra bubble modes for more accurate mechanical bending computations.
- "h11" ("h11xy" or "h11xyz") shape functions (only for hexahedral elements) identical to order one "h1" ("h1xy" or "h1xyz") shape functions except that there are three extra bubble modes for more accurate mechanical bending computations.
- "one" ("onexy" or "onexyz") shape functions on an n dimensional geometry have a single shape function equal to a constant one on n dimensional elements and have no shape functions on lower dimensional elements.

`field(std::string fieldtypename, const std::vector<int> harmonicnumbers)`

mesh mymesh("disk.msh");
field v("h1", {1,4,5,6});
field v4 = v.harmonic(4);

Consider the infinite Fourier series of a field that is periodic in time:
$v(x,t) = V_1 + V_2 \sin(2\pi f_o t) + V_3 \cos(2\pi f_o t) + V_4 \sin(2 \cdot 2\pi f_o t) + V_5 \cos(2 \cdot 2\pi f_o t) + V_6 \sin(3 \cdot 2\pi f_o t) + ...$
where t is the time variable, x the space variable and f_0 the fundamental frequency of the periodic field. The V_i coeffcients only depend on the space variable, not on the time variable which has now moved to the sines and cosines.

In the example above field v is a *multiharmonic* "h1" type field that includes 4 monoharmonic fields: the V_1, V_4, V_5 and V_6 fields in the truncated Fourier series above. All other harmonics in the infinite Fourier series are supposed equal zero so that field v can be rewritten as:
$v(x,t) = V_1 + V_4 \sin(2 \cdot 2\pi f_o t) + V_5 \cos(2 \cdot 2\pi f_o t) + V_6 \sin(3 \cdot 2\pi f_o t)$. This is the truncated multiharmonic representation of field v (which must be periodic in time).
The third line in the example gets harmonic V_4 from field v. It can then be used like any other field.

```
field(std::string fieldtypename, spanningtree spantree)
field(std::string fieldtypename, const std::vector<int> harmonicnumbers,
spanningtree spantree)
```

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2, top = 3;
spanningtree spantree({sur,top});
field a("hcurl", spantree);
field aharmonic("hcurl", {2,3}, spantree);
```

This adds to the constructors above the spanning tree input argument needed when the field has to be gauged (e.g. for the magnetic vector potential formulation of the magnetostatic problem in 3D).

```
mesh mymesh("disk.msh");
field E("hcurl");
int numcomp = E.countcomponents();
```

This returns the number of components of field E (3 here).

```
int countcomponents(void)
```

```
mesh mymesh("disk.msh");
field E("hcurl");
int numcomp = E.countcomponents();
```

This returns the number of components of field E (3 here).

```
std::vector<int> getharmonics(void)
```

```
mesh mymesh("disk.msh");
field v("h1", {1,4,5,6});
std::vector<int> myharms = v.getharmonics();
```

This returns the harmonics of field v ({1,4,5,6} here).

```
void printharmonics(void)
```

```
mesh mymesh("disk.msh");
field v("h1", {1,4,5,6});
v.printharmonics();
```

Print a string showing the harmonics in the field.

`void setname(std::string name)`

```
mesh mymesh("disk.msh");
field v("h1");
v.setname("v");
```

This gives a name to the field (usefull e.g. when printing expressions including fields).

`void print(void)`

```
mesh mymesh("disk.msh");
field v("h1");
v.setname("v");
v.print();
```

Print the field name ("v" here).

`void setorder(int physreg, int interpolorder)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
v.setorder(vol, 3);
```

Sets interpolation order 3 on region number 1. The default interpolation order is 1.

`void setvalue(int physreg, expression input, int extraintegrationdegree = 0)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
v.setvalue(vol, 12 );
```

Sets the field value on region 1 to expression "12". An extra int argument (e.g. +3 or -1) can be used to increase (or decrease) the default integration order when computing the projection of the expression on field v. Increasing it can give a more accurate computation of the expression but might take longer. The default integration order is the v field order + 2.

`void setvalue(int physreg)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
v.setvalue(vol);
```

Sets the field value on region 1 to 0.

`void setconstraint(int physreg, expression input, int extraintegrationdegree = 0)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1"), w("h1");
v.setconstraint(vol, 12+w*w);
```

Forces the field value (i.e. Dirichlet condition) on region 1 to expression "12+w*w" (this gives 12 until w is set to a non-zero value). An extra int argument (e.g. +3 or -1) can be used to increase (or decrease) the default integration order when computing the projection of the expression on field v. Increasing it can give a more accurate computation of the expression but might take longer. The default integration order is the v field order + 2.

`void setconstraint(int physreg)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
v.setconstraint(vol);
```

Forces the field value (i.e. Dirichlet condition) on region 1 to value 0.

`void setgauge(int physreg)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2, top = 3;
spanningtree spantree({sur,top});
field a("hcurl", spantree);
a.setgauge(vol);
```

This sets a gauge condition on region 'vol'. It must be used e.g. for the magnetic vector potential formulation of the magnetostatic problem in 3D since otherwise the algebraic system to solve is singular. It is only defined for edge shape functions ('hcurl'). Its effect is to constrain to zero all degrees of freedom corresponding to:

- gradient type shape functions
- lowest order edge-based shape functions for all edges on the spanning tree provided

---

`void setdata(int physreg, vectorfieldselect myvec, std::string op = "set")`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1"), w("h1");
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v) );
projection.generate();
vec sol = solve(projection.A(), projection.b());

w.setdata(vol, sol|v);
```

The last line transfers the data corresponding to field v in the solution vector 'sol' to field w on the region number 1.
This only works if v and w are of the same type (here they are both "h1" type). In case v has a higher interpolation order than w the higher order dofs are not transferred to w. In the opposite case the higher order dofs of w are zeroed.
The (optional) third argument makes it possible to either *set* the value of field w with the data in the vector (with "set", the default choice) or *add* the data in the vector to field w (with "add").

`void setdata(int physreg, vec myvec, std::string op = "set")`

```
...
v.setdata(vol, sol);
```

This function is equivalent to v.setdata(vol, sol|v).

`field comp(int component)`

```
mesh mymesh("disk.msh");
field u("h1xyz");
field ux = u.comp(0);
field uy = u.comp(1);
field uz = u.comp(2);
```

This function gets the x, y or z component of a field with subfields.

`field compx(void)`

```
mesh mymesh("disk.msh");
field u("h1xyz");
field ux = u.compx();
```

This function gets the x component of a field with multiple subfields.

`field compy(void)`

```
mesh mymesh("disk.msh");
field  u("h1xyz");
field  uy = u.compy();
```

This function gets the y component of a field with multiple subfields.

`field compz(void)`

```
mesh mymesh("disk.msh");
field  u("h1xyz");
field  uz = u.compz();
```

This function gets the z component of a field with multiple subfields.

`field harmonic(int harmonicnumber)`

```
mesh mymesh("disk.msh");
field  u("h1xyz",  {1,2,3});
field  u2 = u.harmonic(2);
```

This function gets a "h1xyz" type field that is the harmonic 2 of field u.

`field harmonic(const std::vector<int> harmonicnumbers)`

```
mesh mymesh("disk.msh");
field  u("h1xyz");
field  u23 = u.harmonic({2,3});
```

This function gets a "h1xyz" type field that includes the harmonics 2 and 3 of field u.

`field sin(int freqindex)`

```
mesh mymesh("disk.msh");
field  u("h1xyz",  {1,2,3,4,5});
field  us = u.sin(2);
```

This function gets a "h1xyz" type field that is the sin harmonic at 2 times the fundamental frequency in field u, i.e. it is harmonic 4.

`field cos(int freqindex)`

```
mesh mymesh("disk.msh");
field  u("h1xyz",  {1,2,3,4,5});
field  uc = u.cos(0);
```

This function gets a “h1xyz” type field that is the cos harmonic at 0 times the fundamental frequency in field u, i.e. it is harmonic 1.

4.4 The *formulation* object (/src/formulation/formulation.h):

The formulation object holds the terms of the weak formulation of the problem to simulate.

`formulation(void)`

formulation myformulation;

This creates an empty formulation object.

`void operator+=(integration integrationobject)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2;
field v("h1"), vh("h1 ",{1,2,3}), u("h1xyz");
formulation projection;

// Nine valid += calls are listed below

// Basic version:
projection += integral(vol, dof(v)*tf(v) );
projection += integral(vol, 2*dof(v)*tf(v,sur), +1 );
projection += integral(vol, compx(u)*dof(v,sur)*tf(v) - 2*tf(v), +1, 2 );
// Assemble on the mesh deformed by field u:
projection += integral(sur, u, dof(v)*tf(v) - 2*tf(v) );
projection += integral(vol, u, dof(v)*tf(v), -1 );
projection += integral(vol, u, dof(v,sur)*tf(v,sur), +3, 1 );

// Assemble with a call to FFT to compute the 20 first harmonics:
projection += integral(vol, 20, (1-vh)*dof(vh)*tf(vh) + vh*tf(vh) );
projection += integral(sur, 20, vh*vh*dof(vh)*tf(vh) - tf(vh), +3 );
projection += integral(vol, 20, vh*dof(vh,sur)*tf(vh), +1, 2 );
// Same as above but assemble on the mesh deformed by field u:
projection += integral(vol, 20, u, vh*vh*dof(vh)*tf(vh) );
projection += integral(vol, 20, u, vh*vh*dof(vh)*tf(vh) - vh*tf(vh), -1 );
projection += integral(sur, 20, u, dof(vh)*tf(vh,sur), +1, 1 );
```

This adds a term to the formulation. All terms are added together and their sum equals zero.
For the first line in the basic version the term is assembled for unknowns (dof) and test functions (tf) defined on region 'vol' and for an integration on all elements in region 'vol' as well. When no region is specified for the dof or the tf then the element integration region (first argument) is used by default. Otherwise, e.g. on line 2 and 3 the dof or tf region used is the one requested. In other words on line

2 unknowns are defined on region 'vol' but test functions only on region 'sur' while on line 3 it is the opposite.
On line 2 of the basic version an extra int is added at the end compared to line 1. This extra int gives the extra number that should be added to the default integration order to perform the numerical integration in the assembly process. The default integration order is order of the unknown + order of the test function + 2. By increasing the integration order a more accurate assembly can be obtained, at the expense of an increased assembling time.
On line 3 there is another extra int which specifies the 'block number' of the term. The default value is 0. Here it is set to 2. This can be of interest when the formulation is generated since one can choose exactly which block numbers to generate and which ones not.
Line 4 through 6 are similar to the first 3 ones except that an extra argument is added. All calculations done when assembling these 3 terms will be performed on the mesh deformed by field u (possibly on a curved mesh).
The last six lines are similar to the first 6 ones except that an extra int has been added (20 here). When the int is positive an FFT will be called during the assembly and the first 20 harmonics will be computed (the harmonics whose magnitude are below a threshold are disregarded). This must be called when assembling a multiharmonic formulation term that is nonlinear in the time variable.

`int countdofs(void)`

```
...
int numdofs = projection.countdofs();
```

This returns the number of degrees of freedom defined in the formulation.

`void generate(void)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
formulation projection;

projection += integral(vol, dof(v)*tf(v), 0, 2 );
projection += integral(vol, dt(dof(v))*tf(v) - 2*tf(v) );
projection += integral(vol, dtdt(dof(v))*tf(v) );

projection.generate();
```

This assembles all terms in the formulation.

`void generatestiffnessmatrix(void)`

...
projection . generatestiffnessmatrix ();

This assembles only the terms in the formulation which have a dof and that dof has no time derivative applied to it. For multiharmonic formulations it generates all terms.
Here it only generates 'dof(v) $\cdot$ tf(v)'.

`void generatedampingmatrix(void)`

...
projection . generatedampingmatrix();

This assembles only the terms in the formulation which have a dof and that dof has an order one time derivative applied to it (i.e. dt). For multiharmonic formulations it generates nothing.
Here it only generates 'dt(dof(v)) $\cdot$ tf(v)'.

`void generatemassmatrix(void)`

...
projection . generatemassmatrix();

This assembles only the terms in the formulation which have a dof and that dof has an order two time derivative applied to it (i.e. dtdt). For multiharmonic formulations it generates nothing.
Here it only generates 'dtdt(dof(v)) $\cdot$ tf(v)'.

`void generaterhs(void)`

...
projection . generaterhs ();

This assembles only the terms in the formulation which have no dof.
Here it only generates '$-2 \cdot$ tf(v)'.

`void generate(std::vector<int> contributionnumbers)`

...
projection . generate ({0,2});

This generates all terms with block number 0 or 2. Here it means all terms are generated since there are only 2 and 0 (default) block numbers.

`void generate(int contributionnumber)`

...
projection . generate (2);

This generates only the block number 2 (i.e. the first integral term).

`vec b(bool keepvector = false)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v) );
projection.generate();
vec b = projection.b(); // or vec b = projection.b(false);
```

This gives the rhs vector b that was assembled during the 'generate' call. If you select 'true' it means everything that has been assembled during the 'generate' call will be added to what is assembled in another 'generate' call later on. If you select 'false' (default) then all the generated data is no more in the formulation, only in vector b.
All entries in vector b that correspond to Dirichlet constraints are set to the constraint value.

`mat A(bool keepfragments = false)`

```
...
mat A = projection.A(); // or mat A = projection.A(false);
```

This gives the matrix A (of $Ax = b$) that was assembled during the 'generate' call. If you select 'true' it means everything that has been assembled during the 'generate' call will be added to what is assembled in another 'generate' call later on. If you select 'false' then all the generated data is no more in the formulation, only in matrix A.

`vec rhs(bool keepvector = false)`

```
...
vec rhs = projection.rhs();
```

Does the same as .b().

`mat K(bool keepfragments = false)`

```
...
mat K = projection.K();
```

Gives the stiffness matrix, i.e. the matrix that is assembled with all terms where the unknown (dof) has no time derivative. For multiharmonic formulations this holds everything. Refer to .A() for the boolean argument.

```
mat C(bool keepfragments = false)
```

...
mat C = projection.C();

Gives the damping matrix, i.e. the matrix that is assembled with all terms where the unknown (dof) has an order one time derivative. For multiharmonic formulations C is empty. Refer to .A() for the boolean argument.

```
mat M(bool keepfragments = false)
```

...
mat M = projection.M();

Gives the mass matrix, i.e. the matrix that is assembled with all terms where the unknown (dof) has an order two time derivative. For multiharmonic formulations M is empty. Refer to .A() for the boolean argument.

```
mat getmatrix(int KCM, bool keepfragments = false)
```

...
mat K = projection.getmatrix(0);
mat C = projection.getmatrix(1);
mat M = projection.getmatrix(2);

The first line is equivalent to .K(), the second to .C() and the third to .M(). Refer to .A() for the boolean argument.

4.5 The *intdensematrix* object (/src/intdensematrix.h):

The intdensematrix object stores a row-major array of ints that corresponds to a dense matrix. Only the functions required to use the other objects are detailed.

`intdensematrix(int numberofrows, int numberofcolumns)`

intdensematrix B(2,3);

This creates an all zero matrix with 2 rows and 3 columns.

`intdensematrix(int numberofrows, int numberofcolumns, int initvalue)`

intdensematrix B(2,3, 12);

This creates a matrix with 2 rows and 3 columns. All entries are assigned to value 'initvalue' (12 here).

`intdensematrix(int numberofrows, int numberofcolumns, const std::vector<int> valvec)`

intdensematrix B(2,3, {1,2,3,4,5,6});

This creates a matrix with 2 rows and 3 columns. The entries are assigned to the value of 'valvec'. As an example B at row 0, column 2 is set to 3.

`intdensematrix(int numberofrows, int numberofcolumns, int init, int step)`

intdensematrix B(2,3, 0,1);

This creates a matrix with 2 rows and 3 columns whose values increase by steps of 'step' (first is 'init').

`int countrows(void)`

intdensematrix B(2,3);
int numrows = B.countrows();

This counts the number of rows in the dense matrix (2 here).

`int countcolumns(void)`

intdensematrix B(2,3);
int numcols = B.countcolumns();

This counts the number of columns in the dense matrix (3 here).

`void print(void)`

intdensematrix B(2,3, 0,1);
B.print ();

This prints the matrix values to the console.

`void printsize(void)`

intdensematrix B(2,3);
B. printsize ();

This prints the matrix size to the console.

`int* getvalues(void)`

intdensematrix B(2,3);
int∗ vals = B.getvalues();

This returns an int array pointer that lets you read or write any value to the array (stay in the array to avoid a segmentation fault). The array is deallocated when the matrix goes out of scope.

4.6 The *mat* object (/src/formulation/mat.h):

The mat object holds a sparse algebraic matrix.

`mat(formulation myformulation, intdensematrix rowadresses, intdensematrix coladresses, densematrix vals)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2;
field v("h1");
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v));

intdensematrix adresses(projection.countdofs(), 1, 0, 1);
densematrix vals(projection.countdofs(), 1, 12);

mat A(projection, adresses, adresses, vals);
```

This creates a matrix object whose dof structure is the one in 'projection'. The matrix non zero values are obtained from the input (in format row of value - column of value - value). Here a diagonal matrix is created with value 12 everywhere.

`int countrows(void)`

```
...
int numrows = A.countrows();
```

This counts the number of rows in the matrix.

`int countcolumns(void)`

```
...
int numcols = A.countcolumns();
```

This counts the number of columns in the matrix.

`int countnnz(void)`

```
...
int numnnz = A.countnnz();
```

This counts the number of non zero entries in the matrix.

`void removeconstraints(void)`

...
v. setconstraint (sur);
A.removeconstraints();

This removes the Dirichlet constraint-entries of the matrix. The new matrix has a structure based on a copy of the one defined in formulation 'projection' but without the Dirichlet constraint entries.

`void reuselu(void)`

...
A.reuselu ();

The LU decomposition of the matrix will be reused in mathop::solve.

`Mat getpetsc(void)`

...
Mat petscmat = A.getpetsc();

This outputs the petsc object corresponding to the matrix. It can be used as any other petsc object.

`void print(void)`

...
A.print ();

This prints the matrix size and values.

`mat copy(void)`

...
mat copiedmat = A.copy();

This creates a full copy of the matrix. Only the values are copied (e.g. the LU reuse property is set back to the default no reuse).

`operators + - *`

...
vec b(projection);
vec x(projection);

vec residual = b − A∗x;
mat AA = A∗A;

Any valid +, - and * operation between vectors and matrices is permitted.

4.7 The *mathop* namespace (/src/expression/operation/mathop.h):

The mathop namespace provides a collection of mathematical tools.

```
double getpi(void)
```

double pi = getpi();

This outputs the value of pi.

```
int regionunion(const std::vector<int> physregs)
```

mesh mymesh("disk.msh");
int sur = 2, top = 3;
int surandtop = regionunion({sur, top});

This creates a new physical region that is the union of surfaces 2 and 3.

```
int regionintersection(const std::vector<int> physregs)
```

mesh mymesh("disk.msh");
int sur = 2, top = 3;
int line = regionintersection({sur, top});

This creates a new physical region that is the intersection of surfaces 2 and 3. Here the new region is a line.

```
int regionexclusion(int physreg, int toexclude)
```

mesh mymesh("disk.msh");
int sur = 2, top = 3;
int surnotop = regionexclusion(sur, top);

This creates a new physical region equal to region 2 but with all regions belonging to region 3 removed. Here, since the intersection of regions 2 and 3 is a line, the new region is still a surface but the nodes and edges on the line have been removed. The number of degrees of freedom that the new region will hold is lower than in region 2.

```
void printvector(std::vector<double> input)
void printvector(std::vector<int> input)
```

std::vector<double> v = {2.4, 3.14, −0.1};
printvector(v);

This prints the size of the vector as well as its values.

`expression norm(expression expr)`

expression myvector = array3x1(1,2,3);
norm(myvector).print();

This gives the L2 norm of an expression vector.

`expression normal(int surfphysreg)`

mesh mymesh("disk.msh");
int sur = 2;
normal(sur).write(sur, "normal.pos");

This defines the vector normal to a surface in 3D or a line in 2D. Depending on the face orientation in the mesh the normal is flipped or not.

`void scatterwrite(std::string filename, std::vector<double> xcoords, std::vector<double> ycoords, std::vector<double> zcoords, std::vector<double> compxevals, std::vector<double> compyevals = {}, std::vector<double> compzevals = {})`

std::vector<double> coordx = {0.0, 1.0, 2.0};
std::vector<double> coordy = {0.0, 1.0, 2.0};
std::vector<double> coordz = {0.0, 0.0, 0.0};
std::vector<double> vals = {10.0, 20.0, 30.0};
scatterwrite("values.pos", coordx, coordy, coordz, vals);

This writes to the output .pos file a set of values at given coordinates. If at least one of 'compyevals' or 'compzevals' is not empty then the values saved are vectors and not scalars.

`void setaxisymmetry(void)`

setaxisymmetry();

This call should be placed at the very beginning of the code. After the call everything will be solved assuming axisymmetry (works for 2D meshes in the xy plane only). All equations should be written in their 3D form.
Please note that in order to correctly take into account the cylindrical coordinate change the space derivatives (dx, dy, dz,...) must be applied to the full field before any component is taken. For a 'h1xyz' type field for example 'dz(compz(u))' will not be transformed correctly while 'compz(dz(u))' will.

`void setfundamentalfrequency(double f)`

setfundamentalfrequency(50);

This defines the fundamental frequency (in Hz) required for multiharmonic problems.

`void settime(double t)`

settime(1e−3);

This sets the time variable t, here to 1 ms.

`double gettime(void)`

settime(1e−3);
double gettime();

This gets the value of the time variable t.

`expression t(void)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
v.setconstraint(vol, sin(2*t()));
```

This gives the time variable in form of an expression. The evaluation gives a value equal to gettime().

`field elementsize(int physreg)`

```
mesh mymesh("disk.msh");
int vol = 1;
field volumes = elementsize(vol);
volumes.write(vol, "volumes.pos");
```

This computes the volume/surface/length of every element respectively for a 3D/2D/1D problem. The values are output in a field of type 'one'.

`expression dx(expression input)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
dx(v).write(vol, "dxv.pos");
```

This is the x space derivative.

`expression dy(expression input)`

...
dy(v).write(vol, "dyv.pos");

This is the y space derivative.

`expression dz(expression input)`

...
dz(v).write(vol, "dzv.pos");

This is the z space derivative.

`expression dt(expression input)`

...
setfundamentalfrequency(50);
field vmh("h1", {2,3});
dt(vmh).write(vol, "dtv.pos");
dt(dt(vmh)).write(vol, "dt−dtv.pos");

This is the first order time derivative.

`expression dtdt(expression input)`

...
setfundamentalfrequency(50);
dtdt(vmh).write(vol, "dtdtv.pos");

This is the second order time derivative.

`expression sin/cos/tan/asin/acos/atan/abs/sqrt/log10(expression input)`

expression expr1 = sin(2);
expression expr2 = cos(2);
expression expr3 = tan(2);
expression expr4 = asin(0.5);
expression expr5 = acos(0.5);
expression expr6 = atan(0.5);
expression expr7 = abs(−2);
expression expr8 = sqrt(2);
expression expr9 = log10(2);

This is the sin/cos/tan/asin/acos/atan/abs/sqrt/log10 function.

`expression pow(expression base, expression exponent)`

expression expr = pow(2, 2);

This is the power expression (^ notation is not supported).

`expression comp(int selectedcomp, expression input)`

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
comp(0, 2*(u+u)).write(vol, "expr.pos");
```

This returns the selected component of a column vector expression. For a matrix expression the whole row is returned in form of an expression. Select component 0 for the first (x) component, 1 for the second (y) and 2 for the third one (z).

`expression compx(expression input)`

```
...
compx(2*(u+u)).write(vol, "compx.pos");
```

This is equivalent to comp(0, expression).

`expression compy(expression input)`

```
...
compy(2*(u+u)).write(vol, "compy.pos");
```

This is equivalent to comp(1, expression).

`expression compz(expression input)`

```
...
compz(2*(u+u)).write(vol, "compz.pos");
```

This is equivalent to comp(2, expression).

`expression entry(int row, int col, expression input)`

```
...
expression arrayentryrow2col0 = entry(2, 0, u);
```

This gets the (row, col) entry in the vector or matrix expression.

`expression transpose(expression input)`

```
...
expression utransposed = transpose(2*u);
```

This transposes the vector or matrix expression.

`expression inverse(expression input)`

```
expression matexpr(3, 3,   {1,2,3,4,5,6,7,8,9});
expression inversedmat = inverse(matexpr);
```

This gets the inverse of a square matrix.

`expression determinant(expression input)`

```
expression matexpr(3, 3,   {1,2,3,4,5,6,7,8,9});
expression detmat = determinant(matexpr);
```

This gets the determinant of a square matrix.

`expression grad(expression input)`

```
mesh mymesh("disk.msh");
int vol = 1;
field v("h1");
grad(v).write(vol, "gradv.pos");
```

This computes the gradient of a scalar expression.
For vector expressions the gradient is computed for every component.

`expression div(expression input)`

```
...
field u("h1xyz");
div(u).write(vol, "divu.pos");
```

This computes the divergence of a vector expression.

`expression curl(expression input)`

```
...
field u("h1xyz");
curl(u).write(vol, "curlu.pos");
```

This computes the curl of a vector expression.

`expression dof(expression input, int physreg = -1)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2;
```

```
field v("h1");
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v));
projection += integral(vol, dof(v, 1)*tf(v) - 2*tf(v));
```

This declares an unknown field (dof for degree of freedom). The dofs are defined only on region 'physreg', which when not provided is set to the element integration region (here to vol).

`expression tf(expression input, int physreg = -1)`

```
...
projection += integral(vol, dof(v)*tf(v, 1) - 2*tf(v));
```

This declares a test function field. The test functions are defined only on region 'physreg', which when not provided is set to the element integration region (here to vol).

`expression array1x2(expression term11, expression term12) and the like`

```
expression myarray = array2x3(1,2,3,4,5,6);
```

This defines a vector or matrix expression of up to 3×3 size. The array is populated in a row major way. In the example the first row is (1,2,3) and the second row is (4,5,6).

`vec solve(mat A, vec b)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2;
field v("h1");
v.setconstraint(sur);
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v));

vec sol = solve(projection.A(), projection.b());
```

This solves an algebraic problem with a (possibly reused) LU decomposition by calling the parallel mumps solver via petsc.

`expression strain(expression input)`

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
expression engstrain = strain(u);
engstrain.print();
```

This defines the (linear) engineering strains in Voigt form (ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, γ_{yz}, γ_{xz},γ_{xy}).

`expression greenlagrangestrain(expression gradu)`

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
expression glstrain = greenlagrangestrain(grad(u));
glstrain.print();
```

This defines the (nonlinear) Green-Lagrange strains in Voigt form (ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, γ_{yz}, γ_{xz},γ_{xy}).

`expression predefinedelasticity(expression dofu, expression tfu, expression Eyoung, expression nupoisson, std::string myoption = "")`

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
formulation elasticity ;
elasticity += integral(vol, predefinedelasticity (dof(u), tf(u), 150e9, 0.3));
elasticity += integral(vol, -2330*dtdt(dof(u))*tf(u));
```

This defines a classical isotropic linear elasticity formulation. Field u is the mechanical displacement. Expression 'Eyoung' is Young's modulus while 'nupoisson' is Poisson's ratio. The second term in the formulation is the inertia term.
In 2D the option string must be either set to "planestrain" or to "planestress" for respectively a plane strain or plane stress assumption.

`predefinedelasticity(expression dofu, expression tfu, expression hookesmatrix, std::string myoption = "")`

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
// It is enough to only provide the lower triangular part of Hooke's matrix (symmetric):
expression H(6,6,{195e9, 36e9,195e9, 64e9,64e9,166e9, 0,0,0,80e9, 0,0,0,0,80e9, 0,0,0,0,0,51e9});
formulation elasticity ;
elasticity += integral(vol, predefinedelasticity (dof(u), tf(u), H));
elasticity += integral(vol, -2330*dtdt(dof(u))*tf(u));
```

This extends the previous function to general anisotropic materials.
Hooke's matrix must be provided in Voigt form (ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, γ_{yz}, γ_{xz},γ_{xy}).

```
expression predefinedelasticity(expression dofu, expression tfu, field u,
expression Eyoung, expression nupoisson, expression prestress, std::string myoption = "")
```

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
expression prestress(6,1,{10e6 ,0,0,0,0,0});
formulation elasticity ;
elasticity += integral(vol, predefinedelasticity(dof(u), tf(u), u, 150e9, 0.3, prestress ));
elasticity += integral(vol, -2330*dtdt(dof(u))*tf(u));
```

This defines an isotropic linear elasticity formulation with geometric nonlinearity taken into account (full-Lagrangian formulation using the Green-Lagrange strain tensor). Problems with large displacements and rotations can be simulated with this equation but **strains must always remain small**. Buckling, snap-through and the like or eigenvalues of prestressed structures can be simulated with the above equation in combination with a nonlinear iteration loop.
Field u is the mechanical displacement. Expression 'Eyoung' is Young's modulus while 'nupoisson' is Poisson's ratio. The prestress vector must be provided in Voigt form (σ_{xx}, σ_{yy}, σ_{zz}, σ_{yz}, σ_{xz},σ_{xy}). Set the prestress expression to '0.0' for no prestress.
The second term in the formulation is the inertia term.
In 2D the option string must be either set to "planestrain" or to "planestress" for respectively a plane strain or plane stress assumption.

```
expression predefinedelasticity(expression dofu, expression tfu, field u,
expression hookesmatrix, expression prestress, std::string myoption = "")
```

```
mesh mymesh("disk.msh");
int vol = 1;
field u("h1xyz");
// It is enough to only provide the lower triangular part of Hooke's matrix (symmetric):
expression H(6,6,{195e9, 36e9,195e9, 64e9,64e9,166e9, 0,0,0,80e9, 0,0,0,0,80e9, 0,0,0,0,0,51e9});
expression prestress(6,1,{10e6 ,0,0,0,0,0});
formulation elasticity ;
elasticity += integral(vol, predefinedelasticity(dof(u), tf(u), u, H, prestress ));
elasticity += integral(vol, -2330*dtdt(dof(u))*tf(u));
```

This extends the previous function to general anisotropic materials.
Hooke's matrix and the prestress vector must be provided in Voigt form (ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, γ_{yz}, γ_{xz},γ_{xy}).

```
expression predefinedelectrostaticforce(expression gradtfu, expression E,
expression epsilon)
```

```
mesh mymesh("disk.msh");
int vol = 1, top = 3;
field v("h1"), u("h1xyz");
formulation elasticity ;
elasticity += integral(vol, predefinedelasticity (dof(u), tf(u), 150e9, 0.3));
elasticity += integral(vol, predefinedelectrostaticforce (grad(tf(u,top)), −grad(v), 8.854e−12));
```

This defines the formulation term to compute the electrostatic forces (obtained with the virtual work principle). The first argument is the gradient of the mechanical displacement test function, the second is the electric field while the third argument is the electric permittivity (must be a scalar).
A region argument should be provided to the test function to compute the force only for the degrees of freedom associated to that specific region (in the example above with *tf(u,top)* the force only acts on surface region 'top'). In any case a correct force calculation requires to include in the **integration domain** all elements in the region where the force acts (in the example above there are none because 'top' is not a volume while the geometry is 3D) **and** in the **element layer around** it (in the example above 'vol' includes all volume elements touching surface 'top').

`expression predefinedmagnetostaticforce(expression gradtfu, expression H, expression mu)`

```
mesh mymesh("disk.msh");
int vol = 1, top = 3;
field phi("h1"), u("h1xyz");
formulation elasticity ;
elasticity += integral(vol, predefinedelasticity (dof(u), tf(u), 150e9, 0.3));
elasticity += integral(vol, predefinedmagnetostaticforce(grad(tf(u,top)), −grad(phi), 4∗getpi()∗1e−7));
```

This defines the formulation term to compute the magnetostatic forces (obtained with the virtual work principle). The first argument is the gradient of the mechanical displacement test function, the second is the magnetic field while the third argument is the magnetic permeability (must be a scalar).
A region argument should be provided to the test function to compute the force only for the degrees of freedom associated to that specific region (in the example above with *tf(u,top)* the force only acts on surface region 'top'). In any case a correct force calculation requires to include in the **integration domain** all elements in the region where the force acts (in the example above there are none because 'top' is not a volume while the geometry is 3D) **and** in the **element layer around** it (in the example above 'vol' includes all volume elements touching surface 'top').

4.8 The *mesh* object (/src/mesh/mesh.h):

The mesh object holds the information of the finite element mesh of the geometry.

`mesh(std::string filename, int verbosity = 1)`

```
mesh mymesh("disk.msh");
```

This creates the mesh object based on the 'disk.msh' mesh file created by GMSH.

`mesh(std::vector<shape> inputshapes, int verbosity = 1)`

```
int faceregionnumber = 1, lineregionnumber = 2;
shape quadface("quadrangle", faceregionnumber, {0,0,0, 1,0,0, 1,1,0, 0,1,0}, {10,6,10,6});
shape leftline = quadface.getsons()[3];
leftline.setphysicalregion(lineregionnumber);
mesh mymesh({quadface, leftline});
mymesh.write("quadmesh.msh");
```

This creates the mesh object based on the quadrangle shape and its left side line.

`void load(std::string filename, int verbosity = 1)`

```
mesh mymesh;
mymesh.load("disk.msh");
```

This loads the 'disk.msh' mesh file created by GMSH.

`void load(std::vector<shape> inputshapes, int verbosity = 1)`

```
int faceregionnumber = 1, lineregionnumber = 2;
shape quadface("quadrangle", faceregionnumber, {0,0,0, 1,0,0, 1,1,0, 0,1,0}, {10,6,10,6});
shape leftline = quadface.getsons()[3];
leftline.setphysicalregion(lineregionnumber);
mesh mymesh;
mymesh.load({quadface, leftline});
mymesh.write("quadmesh.msh");
```

This loads the quadrangle shape and its left side line to the mesh.

`void write(std::string filename, int verbosity = 1)`

```
mesh mymesh("disk.msh");
mymesh.write("disk2.msh");
```

This writes the mesh in object 'mymesh' to file 'disk2.msh'.

`void shift(double x, double y, double z)`

```
mesh mymesh("disk.msh");
mymesh.shift(1.0,  2.0,  3.0);
```

This translates the mesh in object 'mymesh' by 1, 2 and 3 respectively in the x, y and z direction.

`void rotate(double ax, double ay, double az)`

```
mesh mymesh("disk.msh");
mymesh.rotate(20, 60, 90);
```

This rotates the mesh in object 'mymesh' by 20, 60 and 90 degrees respectively around the x, y and z axis.

`int getmeshdimension(void)`

```
mesh mymesh("disk.msh");
int  dim = mymesh.getmeshdimension();
```

This returns the dimension of the highest dimension element in the mesh in object 'mymesh'.

4.9 The *parameter* object (/src/expression/parameter.h):

The parameter object can hold different expression objects on different geometric regions.

`parameter(void)`

mesh mymesh("disk.msh");
parameter E;

This creates parameter E (not yet defined).

`parameter(int numrows, int numcols)`

mesh mymesh("disk.msh");
parameter E(3,3);

This creates a 3 by 3 matrix parameter E (not yet defined).

`int countrows(void)`

mesh mymesh("disk.msh");
parameter E(3,3);
int numrows = E.countrows();

This returns the number of rows in the parameter.

`int countcolumns(void)`

mesh mymesh("disk.msh");
parameter E(3,3);
int numcols = E.countcolumns();

This returns the number of columns in the parameter.

`parameterselectedregion operator|(int physreg)`

mesh mymesh("disk.msh");
int vol = 1;
parameter E;
E|vol = 150e9;

This sets the parameter expression on region 1. Since surface region 2 and 3 are part of volume region 1 in "disk.msh" the parameter is also defined on these two surface regions.

`void print(void)`

```
mesh mymesh("disk.msh");
int  vol = 1;
parameter E;
E|vol = 150e9;
E.print();
```

This prints information on the parameter.

4.10 The *shape* object (/src/geometry/shape.h):

The shape objects are meshed geometric entities. The mesh created based on shapes can be written in .msh format (check the 'mesh' object for that) at any time for visualization in GMSH (it might be needed to change the 'color' and 'visibility' options in the menu 'all mesh options' of GMSH).

`shape(void)`

shape myshape;

This creates an empty shape object.

`shape(std::string shapename, int physreg, std::vector<double> coords)`

int pointphysicalregion = 1, linephysicalregion = 2;
shape mypoint("point", pointphysicalregion, {1.2, 1.5, −0.2});
shape myline("line", linephysicalregion, {0,0,0, 0.5,0.5,0, 1,1,0, 1.5,1,0, 2,1,0});
mesh mymesh({myline, mypoint});
mymesh.write("meshed.msh");

This constructor can be used to create:

- a point at given x, y and z coordinates (1.2,1.5,-0,2)
- a line going through a list of nodes (0,0,0), (0.5,0.5,0),... whose x, y and z coordinates are provided

A physical region number is also provided to easily have access to every geometric region of interest in the finite element simulation.

`shape(std::string shapename, int physreg, std::vector<double> coords, int nummeshpts)`

int linephysicalregion = 1, arcphysicalregion = 1;
shape myline("line", linephysicalregion, {0,0,0, 1,−1,1}, 10);
shape myarc("arc", arcphysicalregion, {1,0,0, 0,1,0, 0,0,0}, 8);
mesh mymesh({myline, myarc});
mymesh.write("meshed.msh");

This constructor can be used to create:

- a straight line between the first (0,0,0) and last point (1,-1,1) provided
- a circle arc between the first point (1,0,0) and the last point (0,1,0) whose center is (0,0,0)

The 'nummeshpts' argument corresponds to the total number of nodes in the meshed shape.

`shape(std::string shapename, int physreg, std::vector<double> coords, std::vector<int> nummeshpts)`

```
int quadphysicalregion = 1;
shape myquadrangle("quadrangle", quadphysicalregion, {0,0,0, 1,0,0, 1,1,0, 0,1,0}, {6,8,6,8});
mesh mymesh({myquadrangle});
mymesh.write("meshed.msh");
```

This constructor can be used to create a general quadrangle with a structured mesh. The 'coords' argument provides the x, y and z coordinates of its 4 corner nodes (0,0,0), (1,0,0), (1,1,0) and (0,1,0). The 'nummeshpts' argument gives the total number of nodes to mesh each of the 4 contour lines.

`shape(std::string shapename, int physreg, std::vector<shape> subshapes, int nummeshpts)`

```
int linephysicalregion = 1, arcphysicalregion = 1;
shape point1("point", −1, {0,0,0});
shape point2("point", −1, {1,0,0});
shape point3("point", −1, {0,1,0});
shape point4("point", −1, {1,−1,1});
shape myline("line", linephysicalregion , {point1, point4}, 10);
shape myarc("arc", arcphysicalregion , {point2, point3, point1}, 8);
mesh mymesh({myline, myarc});
mymesh.write("meshed.msh");
```

This constructor can be used to create:

- a straight line between the first (0,0,0) and last point (1,-1,1) provided
- a circle arc between the first point (1,0,0) and the last point (0,1,0) whose center is (0,0,0)

The 'nummeshpts' argument corresponds to the total number of nodes in the meshed shape.

`shape(std::string shapename, int physreg, std::vector<shape> subshapes, std::vector<int> nummeshpts)`

```
int quadphysicalregion = 1;
shape point1("point", −1, {0,0,0});
shape point2("point", −1, {1,0,0});
shape point3("point", −1, {1,1,0});
shape point4("point", −1, {0,1,0});
shape myquadrangle("quadrangle", quadphysicalregion, {point1, point2, point3, point4}, {6,8,6,8});
mesh mymesh({myquadrangle});
mymesh.write("meshed.msh");
```

This constructor can be used to create a general quadrangle with a structured mesh. The 'subshapes' argument provides its 4 corner point shapes.
The 'nummeshpts' argument gives the total number of nodes to mesh each of the 4 contour lines.

`shape(std::string shapename, int physreg, std::vector<shape> subshapes)`

```
int quadphysicalregion = 1;
shape line1(" line ", -1, {-1,-1,0, 1,-1,0}, 12);
shape arc2("arc ", -1, {1,-1,0, 1,1,0, 0,0,0}, 18);
shape line3(" line ", -1, {1,1,0, -1,1,0}, 12);
shape line4(" line ", -1, {-1,1,0, -1,-1,0}, 18);
shape myquadrangle("quadrangle", quadphysicalregion, {line1, arc2, line3 , line4 });
mesh mymesh({myquadrangle});
mymesh.write("meshed.msh");
```

This constructor can be used to create a general quadrangle with a structured mesh. The 'subshapes' argument provides its contour shapes (clockwise or anti-clockwise).

`shape(std::string shapename, int physreg, std::vector<double> centercoords, double radius, int nummeshpts)`

```
int diskphysicalregion = 1;
shape mydisk("disk", diskphysicalregion , {1,0,0}, 2, 40);
mesh mymesh({mydisk});
mymesh.write("meshed.msh");
```

This constructor creates a radius 2 disk (with a structured mesh) centered around coordinates (1,0,0). The 'nummeshpts' argument corresponds to the total number of nodes in the contour circle of the disk. Because the disk has a structured mesh the number of mesh nodes must be a multiple of 4.

`shape(std::string shapename, int physreg, shape centerpoint, double radius, int nummeshpts)`

```
int diskphysicalregion = 1;
shape centerpoint("point ", -1, {1,0,0});
shape mydisk("disk", diskphysicalregion , centerpoint , 2, 40);
mesh mymesh({mydisk});
mymesh.write("meshed.msh");
```

This constructor creates a radius 2 disk (with a structured mesh) centered around point 'centerpoint'. The 'nummeshpts' argument corresponds to the total number of nodes in the contour circle of the disk. Because the disk has a structured mesh the number of mesh nodes must be a multiple of 4.

`void setphysicalregion(int physreg)`

```
int quadphysicalregion = 1, linephysicalregion = 2;
shape myquadrangle("quadrangle", quadphysicalregion, {0,0,0, 1,0,0, 1,1,0, 0,1,0}, {6,8,6,8});
shape myline = myquadrangle.getsons()[0];
myline.setphysicalregion( linephysicalregion );
mesh mymesh({myquadrangle, myline});
mymesh.write("meshed.msh");
```

This sets the physical region number for a given shape. Subshapes are not affected. The physical region number is used in the finite element simulation to identify a region.

`void deform(expression xdeform, expression ydeform, expression zdeform)`

```
field x("x"), y("y"), z("z");
shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {12,16,12,16});
myquadrangle.deform(0,x,sin(x∗y));
mesh mymesh({myquadrangle});
mymesh.write("meshed.msh");
```

This deforms the shape (and all its subshapes recursively) in the x, y and z direction by a value provided as an expression that can include the x, y and z coordinate fields. When deforming multiple shapes that share common subshapes make sure the subshapes are not deformed multiple times.

`void shift(double shiftx, double shifty, double shiftz)`

```
shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {6,8,6,8});
myquadrangle.shift (1,1,2);
mesh mymesh({myquadrangle});
mymesh.write("meshed.msh");
```

This shifts the shape (and all its subshapes recursively) in the x, y and z direction by a double value. When shifting multiple shapes that share common subshapes make sure the subshapes are not shifted multiple times.

`void rotate(double alphax, double alphay, double alphaz)`

```
shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {6,8,6,8});
myquadrangle.rotate(0,0,45);
mesh mymesh({myquadrangle});
mymesh.write("meshed.msh");
```

This rotates the shape (and all its subshapes recursively) around the x, y and z axis by a double value in degrees. When rotating multiple shapes that share common subshapes make sure the subshapes are not rotated multiple times.

`shape extrude(int physreg, double height, int numlayers)`

int volumephysicalregion = 1;
shape myquadrangle("quadrangle", −1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {6,8,6,8});
shape myvolume = myquadrangle.extrude(volumephysicalregion, 1.4, 6);
mesh mymesh({myvolume});
mymesh.write("meshed.msh");

This outputs a shape of higher dimension that is the extrusion in the z direction of the initial shape. There are 'numlayers' node layers in the extruded mesh. The physical region number of the extruded shape is set to 'physreg'. The extrude function works for 0D, 1D and 2D shapes.

`shape duplicate(void)`

shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {6,8,6,8});
shape otherquadrangle = myquadrangle.duplicate();

This outputs a shape that is the duplicate of the initial shape. All subshapes are duplicated recursively as well but the object equality relations between subshapes are identical between a shape and its duplicate.

`int getdimension(void)`

shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {6,8,6,8});
int dim = myquadrangle.getdimension();
std::cout << dim << std::endl;

This gives the shape dimension (0D, 1D, 2D or 3D).

`std::vector<double> getcoords(void)`

shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {2,2,2,2});
std::vector<double> mycoords = myquadrangle.getcoords();

This returns the coordinates of all nodes in the shape mesh.

`std::string getname(void)`

shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0}, {6,8,6,8});
std::string name = myquadrangle.getname();
std::cout << name << std::endl;

This gives the shape name ("quadrangle" here).

`std::vector<shape> getsons(void)`

```
shape myquadrangle("quadrangle", 1, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0},  {6,8,6,8});
std :: vector<shape> mylines = myquadrangle.getsons();
mylines [0]. setphysicalregion (2);
mylines [1]. setphysicalregion (2);
mylines [2]. setphysicalregion (2);
mylines [3]. setphysicalregion (2);
mesh mymesh(mylines);
mymesh.write("meshed.msh");
```

This outputs a vector containing the direct subshapes of the initial shape. Here it provides the 4 contour lines of the quadrangle.

`int getphysicalregion(void)`

```
shape myquadrangle("quadrangle", 111, {−1,−1,0, 1,−1,0, 1,1,0, −1,1,0},  {6,8,6,8});
int  physreg = myquadrangle.getphysicalregion();
std :: cout << physreg << std::endl;
```

This gives the physical region number of a given shape. The physical region number is used in the finite element simulation to identify a region.

4.11 The *spanningtree* object (/src/mesh/spanningtree.h):

The spanningtree object holds a spanning tree whose edges go through every node of the mesh without forming a loop.

`spanningtree(std::vector<int> physregs)`

mesh mymesh("disk.msh");
int vol = 1, sur = 2, top = 3;
spanningtree spantree({sur,top});

This creates a spanning tree whose edges go through all nodes in the mesh without forming a loop. The tree growth starts on the physical regions provided as argument. Here the tree is first fully grown on face regions 'sur' and 'top' before being extended everywhere.

`int countedgesintree(void)`

...
std :: cout << spantree.countedgesintree() << std::endl;

This returns the number of edges in the spanning tree.

`void write(std::string filename)`

...
spantree.write("spantree.pos");

This writes the tree to a file for visualization.

4.12 The *vec* object (/src/formulation/vec.h):

The vec object holds a vector, be it the solution vector of an algebraic problem or its right handside.

`vec(formulation formul)`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2;
field v("h1");
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v));

vec b(projection);
```

This creates an all zero vector whose structure and size is the one of formulation 'projection'.

`int size(void)`

```
...
int mysize = b.size();
```

This returns the size of the vector. It is equal to the number of dofs in formulation 'projection'.

`void removeconstraints(void)`

```
...
v.setconstraint(sur);
b.removeconstraints();
```

This removes the Dirichlet constraint-entries of the vector. The new vector has a structure based on a copy of the one defined in formulation 'projection' but without the Dirichlet constraint entries.

`void updateconstraints(void)`

```
...
v.setconstraint(sur,1);
b.updateconstraints();
```

This updates the value of all Dirichlet constraint-entries in the vector.

`void setvalues(intdensematrix adresses, densematrix valsmat, std::string op = "set")`

```
mesh mymesh("disk.msh");
int vol = 1, sur = 2;
field v("h1");
```

```
formulation projection;
projection += integral(vol, dof(v)*tf(v) - 2*tf(v));

vec myvec(projection);

intdensematrix adresses(myvec.size(), 1, 0, 1);
densematrix vals(myvec.size(), 1, 12);

myvec.setvalues(adresses, vals);
// or .setvalues(adresses, vals, "set")
// or .setvalues(adresses, vals, "add")
```

This replaces the values in 'myvec' at the adresses in 'adresses' by the values in 'vals'. With "add" the values are not replaced but added. Here all entries are replaced by value 12.

```
void setvalue(int address, double value, std::string op = "set")
```

...

```
myvec.setvalue(2, 2.32);
```

This is similar to the previous one but sets only a value at a single index.

```
densematrix getvalues(intdensematrix adresses)
```

...

```
densematrix vecvals = myvec.getvalues(adresses);
```

This gets the values in the vector that are at the adresses given in 'adresses'.

```
double getvalue(int address)
```

...

```
double vecval = myvec.getvalue(2);
```

This is similar to the previous one but gets only a value at a single index.

```
Vec getpetsc(void)
```

...

```
Vec petscvec = myvec.getpetsc();
```

This gives the petsc object coresponding to the vector. It can be used like any other petsc object.

```
void print(void)
```

...
myvec.print();

This prints the vector values.

`vec copy(void)`

...
vec copiedvec = myvec.copy();

This creates a full copy of the vector.

`double norm(std::string type = "2")`

...
double mynorm2 = myvec.norm(); // or .norm("2")
double mynorm1 = myvec.norm("1");
double mynorminfinity = myvec.norm("infinity");

This outputs the 1, 2 or infinity norm of the vector (default is the 2 norm).

`operators + - *`

...
vec b(projection);
vec x(projection);
mat A = projection.A();

vec residual = b − A∗x;

Any valid +, - and * operation between vectors and matrices is permitted.

4.13 The *wallclock* object (/src/wallclock.h):

The wallclock object provides an easy way to measure wall execution time.

`wallclock(void)`

wallclock myclock;

This initialises the wall clock object.

`void tic(void)`

wallclock myclock;
myclock.tic ();

This resets the clock.

`double toc(void)`

wallclock myclock;
double timelapsed = myclock.toc();

This returns the time elapsed (in ns).

`void print(std::string toprint = "")`

wallclock myclock;
myclock.print("Time elapsed:"); // or myclock.print()

This prints the time elapsed in the most appropriate format (ns, us, ms or s).
It also prints the message in the argument string (if any).

`void pause(void)`

wallclock myclock;
myclock.pause();
// Do something
myclock.resume();
myclock.print ();

This pauses the clock. The pause() and resume() functions allow to time selected operations in loops.

`void resume(void)`

wallclock myclock;
myclock.pause();

```
for (int i = 0; i < 10; i++)
{
        myclock.resume();
        // Do something and time it
        myclock.pause();
        // Do something else
}
myclock.print();
```

This resumes the clock. The pause() and resume() functions allow to time selected operations in loops.